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Evaluating the Range of Applicability of Existing Models of Stress Distribution in the Neck Forming on Cylindrical Specimens during Tensile Testing

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Abstract. This paper is devoted to the post-processing the results of testing the cylindrical specimens for tension. The tensile test are considered from the point of view of studying the rheological properties of metals and alloys. The main problem in identifying hardening curves from tensile test results is the formation of a neck, inside which the stress state of the material differs from uniaxial. The paper considers theoretical solutions to the problem of stress distribution in the neck region, namely, the models of Bridgman, Davidenkov-Spiridonova and Ostsemin. These models make it possible to calculate equivalent stress values even after specimen deformation localizes in the neck. Using the finite element method, the range of applicability of these models was evaluated. It was shown in the paper that the Ostsemin stress distribution model allows to achieve the best results of the material hardening curves identification regardless of its rheological properties. However, the range of applicability of this model is limited by the maximum strain value of 1.5-2.0. With further deformation of the specimen, an overestimation of the calculated values of equivalent stresses with respect to the proper values is observed.

1. Introduction

The rheological properties of metals and alloys are the dependence of the flow stress on the strain and strain rate, the temperature conditions of a metal forming process and the structural state of a workpiece. Taking into account the principle of minimum energy supplied on the deformation, these properties determine the uniformity of the strain distribution in the volume of the workpiece and, as a result, characterize the operational properties of the finished product [1]. The rheological properties of materials also determine the force, torque and power parameters of any metal forming process, which are important technological factors in the production [2].

There are various types of tests to study the rheological properties of metals and alloys [3]. Among them are tensile tests [4–6], compression [7–10] and torsion tests [11–15], a combined process of compression with torsion, and a number of others [16]. At the same time, one of the simplest testing method from the point of view of the implementation is the method of the tensile testing of cylindrical specimens. However, despite the deceptive simplicity and extremely wide distribution, this testing method is rarely applied to study the materials hardening curves. This is due to the complexity of post-processing the experimental data corresponding to the stage of concentrated deformation of the specimen in the forming neck. In the neck a simple uniaxial stress



state of the material changes into a more complex triaxial state, which corresponds to the inhomogeneous distribution of axial tensile stress and other ones along the diameter d in the minimum cross-section of the specimen (figure 1). This leads to the fact that the equivalent flow stress cannot be found by simply dividing the tensile force P by the cross-sectional area of the specimen. Doing it, you can calculate only the true stress in the neck, which is distributed uniformly along any cross-section of the specimen:

$$\sigma_{true} = \frac{4P}{\pi d^2}. \quad (1)$$

But in order to determine the equivalent (effective) stress, you need to use the correction coefficient K , which takes into account the heterogeneity of the stress distribution in the neck [17]:

$$\sigma_{eq} = \frac{\sigma_{true}}{K}. \quad (2)$$

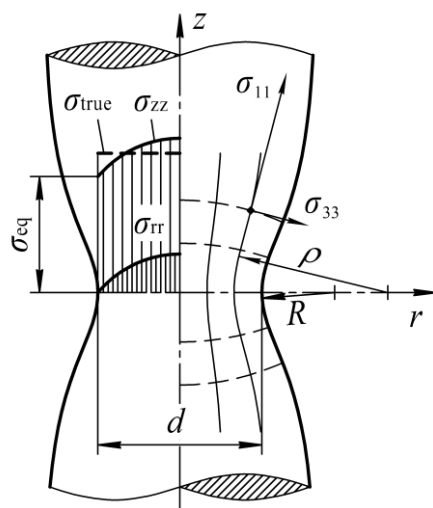


Figure 1. Inhomogeneous stress distribution in the minimum cross-section of the neck:

σ_{zz} – axial tensile stress;

σ_{rr} – radial stress;

σ_{11} – maximum principal stress;

σ_{33} – minimum principal stress.

There is no exact solution to the problem of tensile stress distribution in the neck of the specimen under tension. However, there are various theoretical models of stress distribution. They make it possible to calculate the value of the correction coefficient K based on the geometric parameters of the neck. Among these parameters are the specimen diameter d and the radius of curvature of the neck profile R (figure 1). Bridgman [17], as well as by Davidenkov and Spiridonova [18] developed such models as far back as in the middle of the last century. However, relatively recently Ostsemin proposed new model of stress distribution in the works [19, 20].

It should be noted that the use of any of mentioned above theoretical models represents a certain difficulty associated with assessing the reliability of the hardening curves obtained. In addition, there are no works devoted to the solution of this problem in the research literature. Therefore, the aim of this study is to evaluate the effectiveness of the application of these theoretical models for identifying the hardening curves of metals and alloys according to the results of tensile tests of cylindrical specimens. The addition aim of this study is to determine their applicability depending on the type of the hardening law of a material and its ductility.

2. The models of stress distribution in the neck

The well-known models of stress distribution in the neck of the specimen are based on a number of assumptions. Among the general ones is the assumption that the specimen has rotational symmetry about the z axis. In addition, the specimen is symmetrical in the direction of positive and negative z values relatively to the plane perpendicular to the axis and passing through the minimum cross-section of the neck. The flow stresses satisfy the von Mises condition and at each instant of a time

can be determined by the accumulated strain value. Nevertheless, it is assumed that in the neck section strain changes insignificantly and is uniformly distributed along the specimen radius at each instant of a time. The specimen material is isotropic.

In [17], Bridgman suggested that in the immediate vicinity of the neck the contour of the specimen can be approximated by a tangent circle, and the surface of the principal stresses σ_{33} (dashed lines in figure 1) can be approximated by a sphere. Given these assumptions, he found the stress distribution in the neck section of the specimen:

$$\begin{cases} \sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_{eq} \cdot \ln \frac{a^2 + 2aR - r^2}{2aR}; \\ \sigma_{zz} = \sigma_{eq} + \sigma_{eq} \cdot \ln \frac{a^2 + 2aR - r^2}{2aR}, \end{cases} \quad (3)$$

where r is the radial coordinate and a is the specimen radius in the minimum cross-section of the neck, $a = d/2$.

Based on the equation for calculating tensile stress:

$$P = 2\pi \int_0^a \sigma_{zz} r dr, \quad (4)$$

the correction coefficient K according to the Bridgman model (3) can be found by the formula:

$$K_{Bridgman} = \left(1 + 2 \frac{R}{a}\right) \cdot \ln \left(1 + \frac{a}{2R}\right). \quad (5)$$

Another model of stress distribution proposed by Davidenkov and Spiridonova [18] is based on the assumption that in the minimum neck cross-section the radius of curvature of the maximum principal stress lines σ_{11} varies along the radius r according to the function:

$$\rho = R \frac{a}{r}. \quad (6)$$

Taking into account equation (6), the stress distribution in the neck can be written in the form:

$$\begin{cases} \sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_{eq} \cdot \frac{a^2 - r^2}{2aR}; \\ \sigma_{zz} = \sigma_{eq} + \sigma_{eq} \cdot \frac{a^2 - r^2}{2aR}, \end{cases} \quad (7)$$

and the correction coefficient K can be calculated as follows:

$$K_{Davidenkov} = 1 + \frac{a}{4R}. \quad (8)$$

Ostsemin [19, 20] put forward new assumptions regarding the lines of principal stresses. He considered that the lines of the maximum principal stress σ_{11} are hyperbolas, and the lines of the minimum principal stress σ_{33} are ellipses. Thus, the stress distribution in the neck was obtained:

$$\begin{cases} \sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_{eq} \cdot \ln \frac{a^2 + aR - r^2}{aR}; \\ \sigma_{zz} = \sigma_{eq} + \sigma_{eq} \cdot \ln \frac{a^2 + aR - r^2}{aR}. \end{cases} \quad (9)$$

According to the Ostsemin model, the correction coefficient K can be calculated by the formula:

$$K_{Ostsemin} = \left(1 + \frac{R}{a}\right) \cdot \ln\left(1 + \frac{a}{R}\right). \quad (10)$$

3. Research methodology

The solution of formulated problem is possible only with the use of computer simulation methods. This is because you have ability to compare the material hardening curves calculated using different theoretical models of stress distribution in the neck with the hardening curve that is set in the software during the formulation of the simulation problem. Another advantage of the computer simulation is that the solution to the problem is not limited to the type of the hardening law of a material and its ductility.

Figure 2 presents the hardening curves of the test materials used to simulate the tension of specimens in the Deform-2D software. We examined two materials with hardening, one material had a constant flow stress, and the last one had the effect of softening. We used standard cylindrical specimens (figure 3) corresponding to the Russian standard GOST 1497.

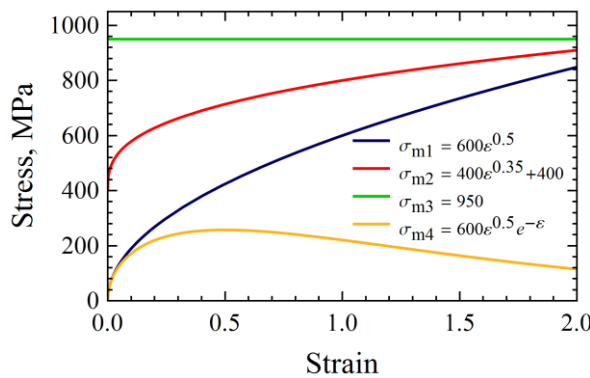


Figure 2. Stress-strain curves of test materials.

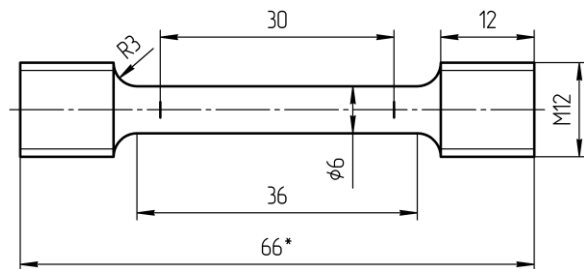


Figure 3. Specimen dimensions.

In order to measure the profile of the forming neck, we used an approach based on the analytical description of all points belonging to the surface of the specimen by the following function:

$$r = r_1 - (r_1 - a) \cdot \left(1 + \frac{z^2}{c}\right)^{-1}, \quad (11)$$

where z and r are the axial and radial coordinates of points on the specimen surface, r_1 is the radius of the gauge length of the specimen corresponding to the beginning of the neck formation and c is the parameter sensitive to differences in material properties. At each simulation step, the coefficients of equation (11) were found by the least-squares deviation method, while the coordinates of all points on the gauge length of the specimen were used for fitting.

From equation (11), we obtained the formula for calculating the radius of curvature of the neck:

$$R = \frac{c}{2 \cdot (r_1 - a)}. \quad (12)$$

It should be noted that described approach is not effective at all simulation steps. For example, for the test material m2 (you can see its hardening law on figure 2) with the total specimen extension of 7.5 mm, the accuracy of the neck profile fitting is 98.3% (figure 4a). And with the extension of 10 mm corresponding to the strain value of 2.44, the fitting accuracy decreases to 88.3% (figure 4b). As you can see, the largest error appears in the center of the neck. This leads to inaccurate calculation of the radius of curvature of the neck R . To increase the accuracy of the post-processing, we applied the

procedure in which the number of taken into account points on the surface was limited iteratively. In each iteration, starting from the second one, in order to fit the neck profile we took only those points corresponding to the specimen surface that were located between the inflection points of the function (11) with the coefficients found in the previous iteration. Practice has shown the convergence of this algorithm at 3-4 iterations. Figure 4c shows an example of using an iterative procedure for the same specimen with the extension of 10 mm. As you can see, the fitting accuracy increases up to 96.0%.

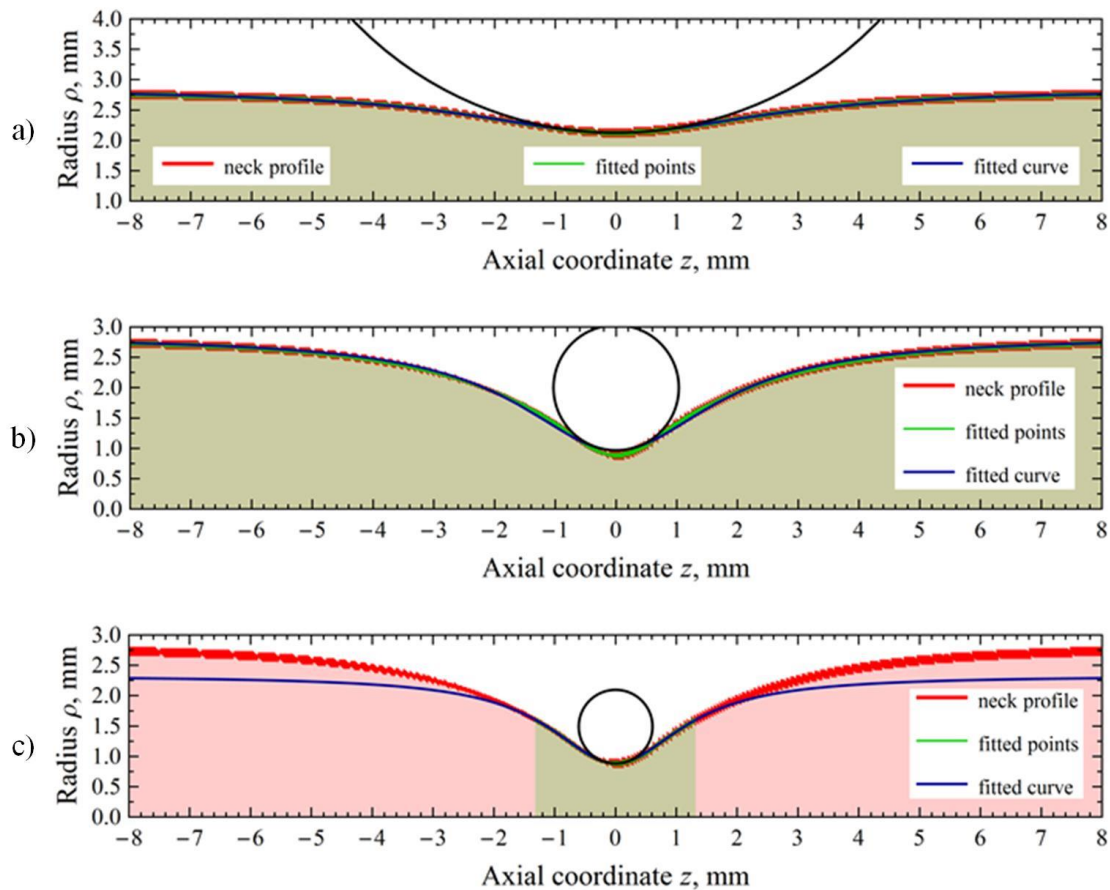


Figure 4. The results of fitting the neck profile for the test material m2: (a) extension of 7.5 mm; (b) extension of 10 mm; (c) extension of 10 mm, iteration procedure was used.

Having determined at each simulation step the values of the radius of the neck curvature R , the radius of the specimen a , and also the tensile force P , using the equations (5), (8) and (10), we calculated the values of the correction coefficients K according to the Bridgman, Davidenkov-Spiridonova and Ostsemin models. Then, using equations (1) and (2), the true and equivalent stresses were determined depending on the strain values. We calculated the last ones according to the equations:

$$\varepsilon_{eq} \approx \varepsilon_{true} = 2 \ln \frac{d_0}{2a}, \quad (13)$$

where d_0 is the initial diameter of the gauge length of the specimen, $d_0 = 6$ mm.

4. Results

Figure 5 presents the results of the calculation of the hardening curves of the test materials in comparison with the corresponding hardening curves that were set during formulation of the finite element simulation problems. A comparative analysis of the simulation results shows that taking into account the inhomogeneity of the stress distribution in the neck can significantly increase the accuracy of the

hardening curves identification, while post-processing the results of real tensile tests. As you can see from figure 5, Ostsemin stress distribution model provides the best result. However, the hardening curve identification remains its high accuracy only up to the strain values of approximately 1.5-2.0. With values of the strain greater than 2, a noticeable overestimation of equivalent stresses relative to a proper hardening curve is observed. The type of the material hardening law has no significant effect on the study of rheological properties.

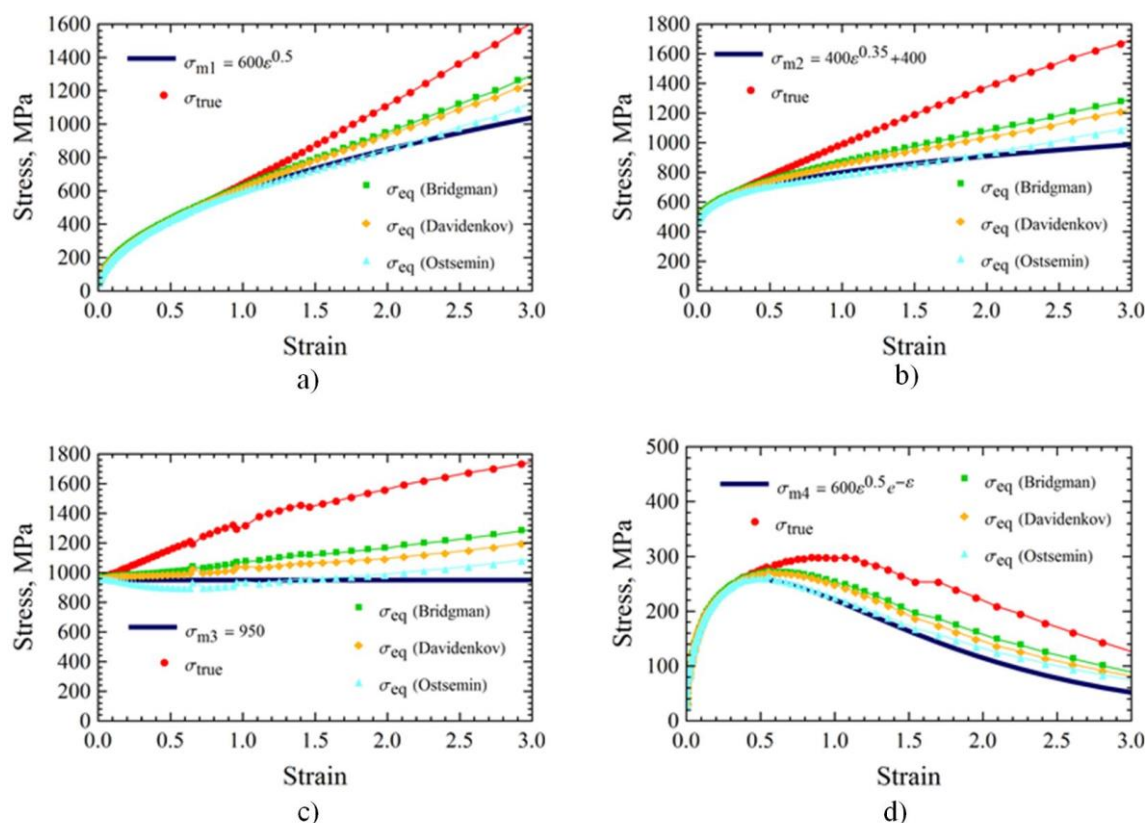


Figure 5. The comparison of the stress-strain curves of the test materials m1 (a), m2 (b), m3 (c) and m4 (d) obtained using different models of stress distribution in the minimum cross-section of the neck.

5. Conclusions

In the work, known models of stress distribution in the neck of a cylindrical specimen under tension are considered. The computer simulation of tensile tests showed that the Ostsemin model allows to achieve the best results of the material hardening curves identification regardless of its rheological properties. However, the range of applicability of this model is limited by the maximum strain value of 1.5-2.0. With further deformation of the specimen, an overestimation of the calculated values of equivalent stresses with respect to the proper values is observed. This phenomenon may be associated with an inhomogeneous distribution of strain over the neck cross-section and should be studied in the course of further researches.

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